Lab 14 - Simple Harmonic Motion and Oscillations on an Incline

I. Introduction/Theory

The purpose of this lab is to measure the period of oscillation of a spring and mass system on an incline at different angles and compare it to the theoretical value. For a mass attached to a spring, the theoretical period of oscillation is given by

$$T = 2\pi \sqrt{\frac{m}{k}}$$

where T is the time for one complete back-and-forth motion, m is the mass that is oscillating, and k is the spring constant.

According to Hooke’s Law, the force exerted by the spring is proportional to the distance the spring is compressed or stretched, \(F = -kx\), where k is the proportionality constant or spring constant. The spring constant can be experimentally determined by applying different forces to stretch the spring different distances. When the force is plotted versus distance, the slope of the resulting straight line is equal to k.
II. Equipment

Dynamics Cart with Mass (ME9430)
Springs (2, one strong and one medium)
Base and Support Rod
Mass Set (100 g to 500 g)
Stopwatch
Mass Balance
Meter Stick

III. Procedure/Data

A. Measurements to Find the Theoretical Period

The purpose of this section is to predict the theoretical period of oscillations, \( T = \frac{2\pi}{\sqrt{k}} \), which are experimentally measured in the next section. The factor needed in this section is the spring constant, \( k \). The spring constant is determined by Hooke’s Law, \( F = -kx \). The force in the Hooke’s Law equation is determined by the ‘inclined’ force of the weight of the mass (\( F = mg \sin(\theta) \)). The displacement, \( x \), is directly measured. The spring constant is thus determined from the slope of a line fit to \( F \) vs. \( x \). Thus, the theoretical period is then determined by the equation above.

1. Use the balance to find the mass of the cart, record this value at the top of Table 1.
2. Set the cart on the track and attach the stiff spring to one end of the cart by inserting the end of the spring in the hole provided in the cart. Then attach the other end of the spring to the end of the track (See Figure 1). Verify/tighten the fixed end stop. Verify/tighten the fixed end stop as necessary during lab.
3. Incline the track by raising the end of the track that has the spring attached. As the end of the track is raised the spring will stretch. Keep the angle of inclination of the track small enough so the spring is not stretched more than 1-2 times the springs unstretched length. Measure this angle and record it at the top of Table 1. The angle of inclination, \( \theta \), should be 20-30°.
4. Record the equilibrium position of the cart at the top of Table 1.
5. Add mass to the cart and record the new position. Repeat this for a total of 5 different masses, being careful not to over-stretch the spring.
6. Repeat steps 2, 3, 4, and 5 for the medium spring. Be careful not to over stretch the spring. Remove masses from the cart at completion of this section.

Mass of Cart = ______________________

Angle of Incline = ______________________

Equilibrium Position (stiff spring) = ______________________

Equilibrium Position (medium spring) = ______________________
Table 1

<table>
<thead>
<tr>
<th>Stiff Spring</th>
<th>Added Mass</th>
<th>Position</th>
<th>Displacement from Equilibrium</th>
<th>Force $mg \sin \theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100 g</td>
<td></td>
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<tr>
<td></td>
<td>200 g</td>
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<td>300 g</td>
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<td>400 g</td>
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<tr>
<td></td>
<td>500 g</td>
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</tr>
<tr>
<td>Medium Spring</td>
<td>100 g</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>200 g</td>
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<td></td>
<td>500 g</td>
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</tbody>
</table>

B. Measurements to Find the Experimental Period

The purpose of this section is to experimentally measure the period of oscillation of the spring mass system with different springs.

7. Record the angle of incline in Table 2. The angle of inclination, $\theta$, should still be 20-30°. Displace the cart (with medium spring) from equilibrium a specific distance and let it go. Time 3 oscillations and record the time for one oscillation by dividing the total time by 3 in Table 2.

8. Repeat this measurement at least 5 times, varying the initial displacement (amplitude) slightly for each trial.

9. Replace the medium spring with the stiff spring and repeat Steps 7 and 8.

10. Change the angle of the incline. Repeat steps 7, 8, and 9.

11. Start with the stiff spring installed, the cart without added mass, and the inclination returned to near its original value.

   i. Record mass of cart without additional mass at the top of Table 3.
   
   ii. Measure the period of the oscillation and record in Table 3 (once only!).
   
   iii. Increase the mass by 100 g and measure, and record in Table 3 the period of oscillation (once only!).
   
   iv. Increase the mass to 200 g and measure, and record in Table 3 the period of oscillation (once only!).
   
   v. Increase the mass to 300 g and measure, and record in Table 3 the period of oscillation (once only!).
vi. Increase the mass to 400 g and measure, and record in Table 3 the period of oscillation (once only!).

vii. Increase the mass to 500 g and measure, and record in Table 3 the period of oscillation (once only!).

Mass of cart without additional mass ___________________

<table>
<thead>
<tr>
<th>Stiff Spring</th>
<th>Added Mass</th>
<th>Total Mass</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0 g</td>
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<tr>
<td></td>
<td>100 g</td>
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<td>400 g</td>
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<td>500 g</td>
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</tbody>
</table>
IV. Analysis
A. Theoretical Period

1. Using the data in Table 1, calculate the force caused by the additional mass in the cart: \( F = mg \sin \theta \), where \( \theta \) is the angle of incline. On a clean sheet of paper, plot force versus displacement for each spring (label PLOT 1 & 2). Draw the best-fit straight line through the data points and determine the slope of the line. The slope is equal to the effective spring constant, \( k \). Attach the plot to the end of this lab report. NOTE: you can also do these plot with LIN_FIT.MWS.

\[
\begin{align*}
    k_{\text{stiff}} &= \phantom{0000} \pm \phantom{0000} \\
    k_{\text{medium}} &= \phantom{0000} \pm \phantom{0000}
\end{align*}
\]

2. Using the mass of the cart and the spring constant calculate the Theoretical period which would be expected for the data in Table 2 using the theoretical formula.

\[
\begin{align*}
    T_{\text{stiff}} &= \phantom{0000} \pm \phantom{0000} \\
    T_{\text{medium}} &= \phantom{0000} \pm \phantom{0000}
\end{align*}
\]

B. Experimental Period

3. Using the data in Table 2, calculate the average time and uncertainties for each set of oscillations.

4. Does the period vary as the angle is changed?

5. How do the experimental values of period statistically compare with the theoretical values? Comment.

6. What would be the period if the angle, \( \theta \), is 90 degrees?

7. The data in Table 3 contains information of period, \( T \), and total mass, \( m \). The period and mass are
theoretically related by the equation at the beginning of this lab. On a Clean sheet of paper construct a plot of this data (labeled PLOT 3) which is LINEAR! T vs. m is not linear, use \( T = 2\pi \sqrt{\frac{m}{k}} \) to determine a linear relation. This plot should show strong support for the theoretical equation for the period. Once completed this sheet is to be attached to the end of this lab report. NOTE: you can also do these plot with LIN_FIT.MWS.

8. From PLOT 3 calculate the spring constant and statistically compare it to the value of the spring constant found in Analysis step 1.

V. Conclusions (include physical concepts and principles investigated in this lab, independent of your experiments success, and summarize without going into the details of the procedure.)