Lab 10 - Conservation of Momentum & Projectile Motion

Name __________________________________________
Partner’s Name __________________________________________

I. Introduction/Theory

This experiment demonstrates the conservation of momentum during a collision, and distinguishes between the conservation of total energy and the conservation of mechanical energy.

A spring "gun" hurls a sphere in a horizontal trajectory so that it hits a catcher mounted on a rod attached to a pivot. This apparatus is called a ballistic pendulum. The catcher traps the sphere in an inelastic collision so that its momentum may be conserved. The catcher plus sphere then has a velocity that makes them rotate in a vertical plane about the pivot until the potential energy becomes equal to that of the kinetic energy after the collision. The pivot has a built-in clutch that holds the pendulum at its highest point and allows an easy measurement of the potential energy increase. While the mechanical energy (kinetic plus potential) after the collision is conserved, the mechanical energy during the collision is not conserved. A test of these theoretical principles will be made by performing two related experiments and comparing the results.

The first experiment measures the increase in potential energy of both the catcher and the sphere as a result of the collision, and uses the conservation of energy and momentum to determine the initial velocity of the sphere just before it strikes the catcher. The second experiment independently finds the initial velocity of the sphere by making a kinematic measurement of the horizontal distance traveled for a particular vertical height when the sphere acts as a projectile. If Newtonian theory is correct, these two initial velocities should agree within experimental uncertainty/error.

II. Equipment

Ballistic Pendulum Set
Meter Stick
Ruler
Scale (100 g capacity)
C-clamp

III. Procedure/Data

Initial Setup:
1. Position the assembled ballistics pendulum at the edge of a table or lab bench. Adjust the feet up or down on the bottom of the base until the bubble in the level, located near the post, indicates that the pendulum base is perfectly level. Clamp the base to the tabletop to prevent accidental movement.

CAUTION: Use care when operating this device. Do not stand or place hands or body in the path of the projectile.

2. Adjustment of the sphere velocity can be made by turning the sleeve on the horizontal shaft which is located on the end of the trigger assembly furthest from the pendulum post assembly. Rotation of the sleeve changes the tension on the spring inside the trigger assembly and produces a noticeable effect only after several complete turns. It is easier to turn the adjustment sleeve before the trigger assembly is in the “cocked” position. The adjustment sleeve should not be changed until both parts of the experiment are completed.

3. In the “un-cocked” position, rotate the sleeve clockwise to maximize the tension in the spring when “cocked”.


Part 1: The Pendulum Experiment

In this first of two experiments to find the initial velocity of the sphere, the sphere will be projected into the catcher and cause them both to rotate about the pivot. The sphere of mass $m$ has a velocity $v$ immediately before it reaches the catcher. The sphere then collides inelastically with the catcher (mass $M$) to give the combined masses a horizontal velocity $V$. During the collision momentum is conserved, but mechanical energy (the sum of kinetic and potential) is not. Momentum is conserved because no external (horizontal) forces act on the system. Mechanical energy is not conserved because the collision is inelastic. Thus, the kinetic energy of the sphere before the collision will not be equal to the potential energy of the sphere and catcher at their highest point after the collision. However, the kinetic energy of the sphere and catcher immediately after the collision will be equal to their potential energy at the highest point of their arc. If $h$ is the vertical height to which the sphere and catcher rise as a result of the collision, then the equations governing this experiment are:

\[ mv = (m + M)V \]  
(conservation of momentum)

and

\[ \frac{1}{2}(m + M)V^2 = (m + M)gh \]  
(conservation of energy after the collision).

Where $v$ is the initial velocity of the ball by itself and $V$ is the initial velocity of the ball and catcher together. Solving the first equation for $v$ and substituting into the second gives the following equation for the initial velocity $v$ of the sphere:

\[ v = \frac{m + M}{m} \sqrt{2gh} \]

The procedure to determine the values of the parameters in the right hand side of this equation is described below. The mass $m$ of the sphere should be determined on a scale. The effective mass of the catcher (116 g) appears on the rear of the trigger housing assembly since the catcher cannot be removed and measured separately. The center of the mass of the catcher/sphere appears as a red dot on the catcher. The difference between the height of this dot before and after the sphere is projected into the catcher is the increase in height $h$ in the equations above. To use the ballistic pendulum most effectively, proceed as follows:

1. Be sure that the assembly instructions at the beginning of this description have been followed carefully.
2. To fire the sphere, slide the sphere onto the horizontal rod projecting toward the pendulum post assembly and its catcher. Be sure the sphere is firmly seated. The firing mechanism is armed when the sphere and rod are pushed horizontally away from the catcher until the trigger engages. CAUTION: Use care when operating this device. Do not stand or place hands or body in the path of the projectile once the
mechanism is armed.
3. After the mechanism is armed, loosen the knob on top of the post so that the pendulum can swing freely. Push the alignment pin (the small rod with red plastic ends) to the rear. Do not let the pendulum and sphere strike the horizontal rod when resetting as this may deform the rod end.
4. **IMPORTANT:** To insure consistently accurate readings the next instruction must be followed exactly. With the top knob loose, guide the pendulum until it touches the sphere. Hold it there while tightening the knob on the top of the post. After the knob is tightened, but before letting the pendulum go, bring the alignment pin to the front. Then let the pendulum swing gently back until it touches the alignment pin. At that point, gently pull the pin to the rear. This operation exactly aligns the pendulum in the vertical position and reduces backlash in the roller clutch.
5. Measure and record in Table 1 the vertical distance from the base to the red dot on the catcher.
6. Fire the pendulum by tapping down on the trigger lever located directly above the sphere. The sphere will be propelled forward to hit the pendulum catcher, becoming trapped in the rubber wedges and causing the pendulum to arc upwards. The roller clutch will stop the pendulum at the peak of its arc and prevent it from dropping back down to its starting position.
7. Determine and record in Table 1 the vertical distance from the base to the red dot on the catcher. The difference between this distance and that measured in step 5 is the height h.
8. After measuring, loosen the knob and let the pendulum slowly return to the starting position. **CAUTION:** Do not let the sphere and catcher strike the horizontal rod. To do so may cause deformation of the rod. Repeat steps 2 through 7 until five measurements have been made.
9. Verify/complete Table 1.
10. Measure and record the mass of the sphere. \( m = \) ________________

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Table 1

11. (This step to be completed at the completion of the lab) From the equations given with this lab, find the initial velocity \( v \) of the sphere before it hits the catcher (show work in space below). This is a value determined by assuming that momentum is conserved during the collision and mechanical energy is
conserved after the collision. This value of $v$ will be determined independently as described in the next experiment.

\[ v = \ldots \]

Part 2: Projectile Motion

This is a purely kinematic experiment to determine the initial velocity $v'$ of the sphere. The sphere will be projected horizontally and the horizontal distance it travels before striking the floor will be measured along with the sphere's initial vertical height above the floor. The sphere travels in a parabolic arc with a constant horizontal velocity and a constant vertical acceleration. The sphere is in the air for a time $t$ during which it travels a horizontal distance $R$ given by

\[ R = v't \]

where $v'$ is the initial velocity of the sphere (the prime, $v'$, indicates experiment 2). The time the sphere is in the air is determined by the vertical motion governed by the constant acceleration equation

\[ H = \frac{1}{2}gt^2 \]

where $H$ is the initial distance of the sphere above the floor and $g$ is the gravitational acceleration. The first equation can be solved for $t$ which can be substituted into the second equation and solved for $v'$ to give
The values of $R$ and $H$ can be measured by the following procedure.

1. The ballistic pendulum should be set up at the end of a table as described at the beginning of this guide. The tension adjustment should not be changed between the previous experiment and this one.
2. Rotate the catcher away from the trigger assembly until it is resting on the top of the launcher and out of the launcher's line of fire.
3. Fire the sphere out onto a clear area on the floor to determine approximately where it will fall. **CAUTION:** Use care when operating this device. Do not stand or place hands or body in the path of the projectile once the mechanism is armed.
4. Fire the sphere onto the clear area and determine where the sphere lands. Measure and record in Table 2 the range $R$ of the sphere. **CAUTION:** Use care when operating this device. Do not stand or place hands or body in the path of the projectile once the mechanism is armed.
5. Repeat step 4 until a total of five measures of horizontal distance are recorded from the end of the firing pin.
6. Measure the vertical distance from the bottom of the sphere to the floor when it is on its rod (but not when the firing mechanism is cocked). Determine and record this vertical distance $H$ with uncertainty.

$$H = \underline{_________________} \pm \underline{_________________}$$

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Table 2
7. (This step to be completed at the completion of the lab) From the equations given in this experiment, find the initial velocity $v'$ of the sphere (show work in space below).
IV. Analysis

1. The initial velocity \( v \) and \( v' \) of the sphere and the uncertainty in the velocity \( s_v \) and \( s_{v'} \) can now be calculated for the two experiments. The work for \( v \) and \( v' \) are in previous steps. On a clean sheet of paper that is to be stapled to this lab calculate \( s_v \) and \( s_{v'} \). List the values calculated for \( v, v', s_v, \) and \( s_{v'} \) in the space below. You would expect the two values to agree within the uncertainty of your measurements. Do they? If they do, you have supported the law of conservation of momentum, comment.

2. In addition to the measured errors in the parameter's change in height, \( h \), in the first experiment and \( R \) in the second, a number of assumptions were also made in the equations. The first experiment neglected the friction in the roller bearings and slippage of the clutch; the second experiment neglected air friction. You can think of other assumptions. Make a list and describe how each would effect the calculated values of \( v \) and \( v' \).
3. Earlier it was stated that mechanical energy was not conserved during the collision. Verify this by calculating the velocity $V$ of the sphere and catcher after the collision, and then determining the kinetic energy $\frac{1}{2}(m + M)V^2$ of this combination. Compare that energy with the kinetic energy of the sphere $\frac{1}{2}mv^2$ before the collision. Even though these two energies are quite different, the total energy of the system was conserved. The difference in kinetic energies went into thermal energy (that warmed the sphere & catcher) and sound energy.

V. Conclusions (include physical concepts and principles investigated in this lab, independent of your experiments success, and summarize without going into the details of the procedure.)