Lab 5 – Forces: \( \sum \vec{F} = 0 \)

Name ________________________________

Partner’s Name ________________________________

I. Introduction

The purpose of this experiment is to study the characteristics of Force and the conditions that bring about equilibrium of a rigid object acted upon by several forces. Equilibrium is an important consideration, since unbalanced forces always lead to motion, not a good idea for a bridge or building. The Force Table studies forces in equilibrium and verifies the laws for the summation of forces.

Force is one of the fundamental concepts upon which the subject of mechanics is built. A force is the action of one body on another body which changes or tends to change the state of motion of the body acted on. The idea of force, then, implies the mutual actions of two bodies since one body cannot exert a force on another body unless the second offers a resistance to the first. A force, therefore, never exists alone. Forces always occur in pairs. If a body is acted on by one force only, a change of motion of the body will always take place; but if the body is acted on by two or more forces, it may be held at rest.

Equilibrium requires that the sum of the forces acting at a point is zero. In vector notation the symbol \( \vec{F} \) representing force is shown (commonly in boldface).

\[
\sum_i \vec{F}_i = 0
\]

The summation is made more complex since force is a vector, rather than a scalar quantity. A vector has both magnitude and direction to be considered together. In the Force Table, the forces are provided by gravity acting upon weights and weight holders. The direction of the force vectors is determined by pulleys arranged around an angle scale. Thus both quantities of the force vectors, the magnitude and direction, can be determined.

Theory (This should be a review of vectors!)

Measurable quantities may be classified as either (1) scalar quantities or (2) vector quantities. A scalar quantity has only magnitude. The mass of an object is a scalar quantity. A vector quantity, on the other hand, has both magnitude and direction. For example, to specify completely the velocity of a body, it is necessary to state not only how fast it is traveling but in what direction it is going. Velocity is a vector quantity. Since the weight of a body is the force with which it is attracted by the earth's gravity, weight is a vector quantity. Since weight and mass are different physical concepts, they are not measured in the same physical units. The kilogram (kg) is a unit of mass. The force with which the earth attracts a one kilogram mass is 9.8 Newtons (N).

Figure 1 illustrates one method of representing vector quantities. Assume that the vector quantities in the Figure are forces, but the method applies equally well to all vector quantities. A boldface symbol is used to indicate the magnitude and direction. The arrow A is a vector representing a force. The length of the line \( \overline{oa} \) is drawn to scale to indicate the magnitude of the force. For example, 1 cm might be taken to represent 0.2 N of force. The direction of the line \( \overline{oa} \) indicates the direction of the force and the arrowhead indicates the sense. In this case from left to right. Similarly, the vectors B and C represent other forces.
If two or more forces are concurrent (meet at a common point), they may be replaced by a single force called the resultant. One method of finding the resultant is shown in Figure 1. Having drawn the vectors A and B to scale and in the proper directions, the parallelogram oamb is constructed. The diagonal M of this parallelogram represents the resultant of the forces A and B. The scale previously chosen is used to determine the magnitude of the resultant and a protractor to determine the angle it makes with some chosen direction. Since M may be used to replace A and B it should be obvious that M may be combined with C to find the resultant of A, B, and C. This method is known as the parallelogram method. The resultant of two or more forces is sometimes called the vector sum of the forces.

The graphical method is useful to understand how vectors combine, but it is not the most accurate method of determining the result. Dealing with the forces a pair at a time, the resultant can be obtained from the trigonometric relationships

\[ M = (A^2 + B^2 + 2AB \cos(\beta))^{1/2} \]

and

\[ \tan(\theta) = \frac{A \sin(\beta)}{A + B \cos(\beta)} \]
Since these quantities can be calculated on a hand calculator, the resultant can be obtained more accurately and more rapidly than using the graphical method. If more than two vectors are being combined, it becomes easy to confuse the angles between the vectors, since they are measured relative to each other and not relative to a fixed axis.

Since vectors can be combined without changing the result the components of a vector can be replaced by its components on a pair of fixed axes as shown in Figure 2.

Figure 2

The force $B$ is the resultant of forces $B_x$ and $B_y$ which are its components on the orthogonal axes. The magnitude of these components is given by

$$B_x = B \cos(\beta)$$

and

$$B_y = B \sin(\beta)$$

Each of the vectors being summed together can be treated the same way so that the resultant force can be calculated from the sum of the component forces projected on the $x$ and $y$ axes.

$$R_x = A_x + B_x + C_x + D_x$$

and

$$R_y = A_y + B_y + C_y + D_y$$

The magnitude of the resultant it is

$$R^2 = R_x^2 + R_y^2$$
or more generally, for any number of vectors

\[ R^2 = \left( \sum F_x^2 + \sum F_y^2 \right) \]

and the angle the resultant makes with the x axis is given by

\[ \tan(\phi) = \frac{\sum F_y}{\sum F_x} \]

This discussion has been about force vectors but the conclusions apply equally well to vectors representing velocity, electric fields, fluid flow or other vector fields.

A body is said to be in equilibrium if it has no acceleration. A body that has no acceleration may be either at rest or moving with uniform speed in a straight line. This experiment deals with a body at rest. From Newton's second law it follows that if a body is in equilibrium, the resultant of all the forces acting on it must be zero. If the forces acting on a body have a resultant other than zero the body may be put into equilibrium by adding a force equal and opposite to the resultant force. This force is called the anti-resultant or equilibrant force.

II. Equipment

- Force Table Apparatus
- Ruler
- Scale

III. Procedure

0. On Figure 1, Graphically sum vectors \( \mathbf{A}, \mathbf{B}, \text{and} \ \mathbf{C} \).

Four Unequal Forces

1. Set the Force Table on a supporting table. It is fairly heavy so a sturdy table is needed.
2. Level the Force Table with the adjusting screws on its feet. This adjustment is not critical but the hanging weights should hang vertically.
3. Place the ring over the center post. This will keep everything in place while making adjustments.
4. Attach four of the weight holders and hang them over four of the edge pulleys.
5. Distribute the weights so that the four holders carry 50 g, 100 g, 200 g, and 250 g, respectively.
6. Adjust the positions of the four pulleys until a balance is found. Three of the pulleys will have to be adjusted to center the ring. It will take some time to do this accurately. Don't get discouraged.
7. When nearly balanced, lift the center ring a little and drop it to vibrate the system a little to overcome friction. Make sure that the pulleys are adjusted so that the string passes directly over the index line on the pulley. This will assure that the string is passing straight over the pulley to the center of the table.
8. Have your instructor verify your setup.
9. Tabulate the four angles in Table 2.
10. Interchange the 250 g and 50 g weights and repeat steps 6 through 9.

The scientific international (SI) unit for mass is the kilogram (kg). The weights with the Force Table are calibrated in grams so they must be multiplied by 1kg/1000g. The force is defined by the force produced by the downward pull of gravity on a mass. Its unit is the Newton (N) which is numerically equal to the mass in kg multiplied by 9.8 m/s\(^2\), the acceleration due to gravity. These units have been used in the sample results shown in Table 1.
Weight (kg) | Force(N) | Angle(0) | x component | y component
---|---|---|---|---
.100 | 0.98 | 62.5 | 0.453 | 0.869
.200 | 1.96 | 10.5 | 1.927 | 0.357
.300 | 2.94 | 290.5 | 1.030 | -2.754
.400 | 3.92 | 156.5 | -3.5951 | 0.563
Totals | | | -.185 | .035

Table 1

The sum of the x and y components is almost zero, well within the margin of error for these types of measurements.

These results show that the sum of the x and y components are always zero for an equilibrium adjustment of the forces. The results that you obtain should be similar. It takes a little practice to adjust the pulley positions, but with patience and care good results should be obtained.

IV. Data

1. In Table 1, why aren't the totals of the component forces exactly zero?

2. Complete Table 2.

<table>
<thead>
<tr>
<th>Mass</th>
<th>Force(N)</th>
<th>Angle(0)</th>
<th>x component</th>
<th>y component</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 g</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>100 g</td>
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<tr>
<td>200 g</td>
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<td>250 g</td>
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<td></td>
<td>Totals</td>
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<tr>
<td>Second</td>
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<tr>
<td>50 g</td>
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<td>100 g</td>
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<td>200 g</td>
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<td>250 g</td>
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<tr>
<td></td>
<td>Totals</td>
<td></td>
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</tr>
</tbody>
</table>

Table 2
V. Analysis

1. In Table 2, are the totals of the component forces exactly zero? Why or Why not?

2. What do you think the greatest source of error is?

VI. Conclusion (include physical concepts and principles investigated in this lab, independent of your experiments success, and summarize without going into the details of the procedure.)