

Cosets

Let G be a group and let H be a subset of G . For any $a \in G$, the set $\{ah \mid h \in H\}$ is denoted by aH . Analogously, $Ha = \{ha \mid h \in H\}$ and $aHa^{-1} = \{aha^{-1} \mid h \in H\}$. When H is a subgroup of G , the set aH is called the left coset of H in G containing a , whereas Ha is called the right coset of H in G containing a . In this case, the element a is called the coset representative of aH (or Ha). We use $|aH|$ ($|Ha|$) to denote the number of elements in the set aH (Ha).

Z_4

In GE, double click on Z_4

Click on the Multiplication Table

On the right are two tabs, Subsets and Table

Under the Subsets tab are all Subgroups

Switch to the Table tab, you can now organize the table according to subgroup

*Be sure to switch between the Subsets and Table tabs to see which Subgroup is being used

In the drop-down menu, choose H_1

Notice that the "Separate Cosets By" bar is now accessible

Slide this bar over to the right to separate cosets with gray space

1. What are the distinct cosets for H_1 ?
2. How many cosets are there?

S_3 – Permutations of $\{1, 2, 3\}$

1. What are the elements of S_3 ?
2. How many subgroups are there? What are they?

In GE, open the group S_3 and click on the Multiplication Table

Switch to the Table tab and choose H_1 in the drop down menu

Now that the "Separate Cosets By" bar is accessible, slide the bar over and separate cosets with gray space

3. What are the distinct cosets for H_1 ?
4. Why are some of these partitions not cosets?
5. How many cosets are there for H_1 ? H_2 ? H_3 ? H_4 ?
6. Are these cosets the same?

D_4 – Symmetries of a Square

1. What are the elements of D_4 ?
2. How many subgroups are there? What are they?

In GE, open the group D_4 and click on the Multiplication Table
Switch to the Table tab and choose H_1 in the drop down menu
Now that the “Separate Cosets By” bar accessible, slide the bar over and separate cosets with gray space

3. What is H_8 ? What are its cosets?
4. How many cosets are there for $H_1 - H_5$?
5. Are these cosets the same?
6. How many cosets are in $H_6 - H_8$?
7. Are these cosets the same?

Using these examples, find the left and right cosets of $U(15)$

Take a look at some other multiplication tables and cosets using Group Explorer

- What could be some possible properties of cosets?
- Can you prove these?
- What about the correlation between the size of groups and subgroups?