Homework # 2 (Due Sept 5th)

1. (3 pts) Prove that $2^n + 3^n$ is a multiple of 5 for all odd $n \in N$. (Proof by induction)

2. (3 pts) For each subset of $\Re$, give an upperbound and its supremum, if they exist. Otherwise, write none.

(a) $\left\{\frac{n}{n+1} : n \in N\right\}$;

(b) $\left\{1 - \frac{1}{n} : n \in N\right\}$;

(c) $(-\infty, 4)$.

3. (4 pts) Given nonempty subsets A and B of $\Re$, let C denote the set

$$C = \{x + y : x \in A \textit{ and } y \in B\}.$$

If A and B have supremum, then C has a supremum and

$$\sup C = \sup A + \sup B.$$

(Hint: Use completeness axiom to show that C has a supremum. Let $c = \sup C$, to show $c = a + b$ where $a = \sup A$ and $b = \sup B$, show $c \geq a + b$ and then $c \leq a + b$.)