

**Part 2**  
**Fundamentals of Financial Markets**



# Chapter 3

## What Do Interest Rates Mean and What is Their Role in Valuation?

### Measuring Interest Rates

- Present Value

- Four types of Credit Market Instruments

- Yield to Maturity

  - Global Box: Negative T-Bill Rates? Japan Shows the Way

### The Distinction Between Real and Nominal Interest Rates

### The Distinction Between Interest Rates and Returns

  - Mini-Case Box: With TIPS, Real Interest Rates Have Become Observable in the United States

- Maturity and the Volatility of Bond Returns: Interest-Rate Risk

  - Mini-Case Box: Helping Investors to Select Desired Interest-Rate Risk

- Reinvestment Risk

- Summary

### The Practicing Manager: Calculating Duration to Measure Interest-Rate Risk

- Calculating Duration

- Duration and Interest-Rate Risk

## ■ Overview and Teaching Tips

In my years of teaching financial markets and institutions, I have found that students have trouble with what I consider to be easy material because they do not understand what an interest rate is—that it is negatively associated with the price of a bond, that it differs from the return on a bond, and that there is an important distinction between real and nominal interest rates.

This chapter spends more time on these issues than does any other competing textbook. My experience has been that giving this material so much attention is well rewarded. After putting more emphasis on this material in my financial markets and institutions course, I witnessed a dramatic improvement in students' understanding of portfolio choice and asset and liability management in financial institutions.

An innovative feature of the book that first appears in this chapter is the “Study Guide,” which provides helpful hints to the student, making it easier for him or her to master the material. This section tells the students the key concept needed to master the content and suggests ways in which they can more easily grasp the concept and put it into practice.

Another innovative feature of the textbook is the set of over twenty special applications called, “The Practicing Manager.” These applications introduce students to real-world problems that managers of financial institutions have to solve and make the course both more relevant and exciting to students. They are not meant to fully prepare students for jobs in financial institutions—it is up to more specialized courses such as bank or financial institutions management to do this—but these applications teach them some of the special analytical tools that they will need when they enter the business world.

This chapter contains the Practicing Manager application on “Calculating Duration to Measure Interest-Rate Risk.” The application shows how to quantify interest-rate risk using the duration concept and is a basic tool for managers of financial institutions. For those instructors who do not want a managerial emphasis in their financial markets and institutions course, this and other Practicing Manager applications can be skipped without loss of continuity.

## ■ Answers to End-of-Chapter Questions

1.  $\$2000 = \$100/(1 + i) + \$100/(1 + i)^2 + \dots + \$100/(1 + i)^{20} + \$1000/(1 + i)^{20}$ .
2. You would rather be holding long-term bonds because their price would increase more than the price of the short-term bonds, giving them a higher return.
3. No. If interest rates rise sharply in the future, long-term bonds may suffer such a sharp fall in price that their return might be quite low, possibly even negative.
4. People are more likely to buy houses because the real interest rate when purchasing a house has fallen from 3 percent (=5 percent –2 percent) to 1 percent (=10 percent –9 percent). The real cost of financing the house is thus lower, even though mortgage rates have risen. (If the tax deductibility of interest payments is allowed for, then it becomes even more likely that people will buy houses.)

## ■ Quantitative Problem

1. Calculate the present value of \$1,000 zero-coupon bond with 5 years to maturity if the required annual interest rate is 6%.

**Solution:**  $PV = FV/(1 + i)^n$ , where  $FV = 1000$ ,  $i = 0.06$ ,  $n = 5$   
 $PV = 747.25$  grand prize is

2. A lottery claims its grand prize is \$10 million, payable over 20 years at \$500,000 per year. If the first payment is made immediately, what is this grand prize really worth? Use a discount rate of 6%.

**Solution:** This is a simple present value problem. Using a financial calculator:  
 $N = 20$ ;  $PMT = 500,000$ ;  $FV = 0$ ;  $I = 6\%$ ; Pmts in BEGIN mode.  
Compute PV :  $PV = \$6,079,058.25$

3. Consider a bond with a 7% annual coupon and a face value of \$1,000. Complete the following table:

Years to Maturity	Discount Rate	Current Price
3	5	
3	7	
6	7	
9	7	
9	9	

What relationship do you observe between yield to maturity and the current market value?

**Solution:**

Years to Maturity	Yield to Maturity	Current Price
3	5	\$1,054.46
3	7	\$1,000.00
6	7	\$1,000.00
9	5	\$1,142.16
9	9	\$880.10

When yield to maturity is above the coupon rate, the bond's current price is below its face value. The opposite holds true when yield to maturity is below the coupon rate. For a given maturity, the bond's current price falls as yield to maturity rises. For a given yield to maturity, a bond's value rises as its maturity increases. When yield to maturity equals the coupon rate, a bond's current price equals its face value regardless of years to maturity.

4. Consider a coupon bond that has a \$1,000 per value and a coupon rate of 10%. The bond is currently selling for \$1,150 and has 8 years to maturity. What is the bond's yield to maturity?

**Solution:** To calculate the bond's yield to maturity using a financial calculator:

$$N = 8; PMT = 1000 \times 0.10 = 100; FV = 1000; PV = 1150$$

$$\text{Compute } I : I = 7.44$$

5. You are willing to pay \$15,625 now to purchase a perpetuity which will pay you and your heirs \$1,250 each year, forever, starting at the end of this year. If your required rate of return does not change, how much would you be willing to pay if this were a 20-year, annual payment, ordinary annuity instead of a perpetuity?

**Solution:** To find your yield to maturity, Perpetuity value =  $PMT/I$ .

$$\text{So, } 15625 = 1250/I. I = 0.08$$

The answer to the final part, using a financial calculator:

$$N = 20; I = 8; PMT = 1250; FV = 0$$

$$\text{Compute } PV : PV = 12,272.69$$

7. Property taxes in DeKalb County are roughly 2.66% of the purchase price every year. If you just bought a \$100,000 home, what is the PV of *all* the future property tax payments? Assume that the house remains worth \$100,000 forever, property tax rates never change, and that a 9% discount rate is used for discounting.

**Solution:** The taxes on a \$100,000 home are roughly  $100,000 \times 0.0266 = 2,660$ .

The PV of all future payments =  $2,660/0.09 = \$29,555.55$  (a perpetuity).

8. Assume you just deposited \$1,000 into a bank account. The current real interest rate is 2% and inflation is expected to be 6% over the next year. What nominal interest rate would you require from the bank over the next year? How much money will you have at the end of one year? If you are saving to buy a stereo that currently sells for \$1,050, will you have enough to buy it?

**Solution:** The required nominal rate would be:

$$\begin{aligned} i &= i_r + \pi^e \\ &= 2\% + 6\% = 8\%. \end{aligned}$$

At this rate, you would expect to have  $\$1,000 \times 1.08$ , or \$1,080 at the end of the year. Can you afford the stereo? In theory, the price of the stereo will increase with the rate of inflation. So, one year later, the stereo will cost  $\$1,050 \times 1.06$ , or \$1,113. You will be short by \$33.

9. A 10-year, 7% coupon bond with a face value of \$1,000 is currently selling for \$871.65. Compute your rate of return if you sell the bond next year for \$880.10.

**Solution:**  $R = \frac{C + P_{t+1} - P_t}{P_t} = \frac{70 + 880.10 - 871.65}{871.65} = 0.09$ , or 9%.

10. You have paid \$980.30 for an 8% coupon bond with a face value of \$1,000 that mature in five years. You plan on holding the bond for one year. If you want to earn a 9% rate of return on this investment, what price must you sell the bond for? Is this realistic?

**Solution:** To find the price, solve  $\frac{80 + P_{t+1} - 980.30}{980.30} = 0.09$  for  $P_{t+1}$ .  $P_{t+1} = 988.53$ .

Although this appears possible, the yield to maturity when you purchased the bond was 8.5%. At that yield, you only expect the price to be \$983.62 next year. In fact, the yield would have to drop to 8.35% for the price to be \$988.53.

11. Calculate the duration of a \$1,000 6% coupon bond with three years to maturity. Assume that all market interest rates are 7%.

**Solution:**

Year	1	2	3	Sum
Payments	60.00	60.00	1060.00	
PV of Payments	56.07	52.41	865.28	973.76
Time Weighted PV of Payments	56.07	104.81	2595.83	
Time Weighted PV of Payments	0.06	0.11	2.67	2.83
Divided by Price				

This bond has a duration of 2.83 years. Note that the current price of the bond is \$973.76, which is the sum of the individual “PV of payments.”

12. Consider the bond in the previous question. Calculate the expected price change if interest rates drop to 6.75% using the duration approximation. Calculate the actual price change using discounted cash flow.

**Solution:** Using the duration approximation, the price change would be:

$$\Delta P = -\text{DUR} \times \frac{\Delta i}{1+i} \times P = -2.83 \times \frac{-0.0025}{1.07} \times 973.76 = 6.44.$$

The new price would be \$980.20. Using a discounted cash flow approach, the price is 980.23—only \$.03 different.

Year	1	2	3	Sum
Payments	60.00	60.00	1060.00	
PV of payments	56.21	52.65	871.37	980.23

13. The duration of a \$100 million portfolio is 10 years. \$40 million in new securities are added to the portfolio, increasing the duration of the portfolio to 12.5 years. What is the duration of the \$40 million in new securities?

**Solution:** First, note that the portfolio now has \$140 million in it. The duration of a portfolio is the weighted average duration of its individual securities. Let D equal the duration of the \$40 million in new securities. Then, this implies:

$$12.5 = (100/140 \times 10) + (40/140 \times D)$$

$$12.5 = 7.1425 + 0.2857 D$$

$$18.75 = D$$

The new securities have a duration of 18.75 years.

14. A bank has two, 3-year commercial loans with a present value of \$70 million. The first is a \$30 million loan that requires a single payment of \$37.8 million in 3 years, with no other payments till then. The second is for \$40 million. It requires an annual interest payment of \$3.6 million. The principal of \$40 million is due in 3 years.
- (a) What is the duration of the bank's commercial loan portfolio?
- (b) What will happen to the value of its portfolio if the general level of interest rates increased from 8% to 8.5%?

**Solution:** The duration of the first loan is 3 years since it is a zero-coupon loan. The duration of the second loan is as follows:

Year	1	2	3	Sum
Payment	3.60	3.60	43.60	
PV of Payments	3.33	3.09	34.61	41.03
Time Weighted PV of Payments	3.33	6.18	103.83	
Time Weighted PV of Payments Divided by Price	0.08	0.15	2.53	2.76

The duration of a portfolio is the weighted average duration of its individual securities. So, the portfolio's duration =  $3/7 * (3) + 4/7 * (2.76) = 2.86$

$$\text{If rates increased, } \Delta P = -\text{DUR} \times \frac{\Delta i}{1+i} \times P = -2.86 \times \frac{0.005}{1.08} \times 70,000,000 = -926,852.$$

15. Consider a bond that promises the following cash flows. The required discount rate is 12%.

Year	0	1	2	3	4
Promised Payments		160	170	180	230

You plan to buy this bond, hold it for 2½ years, and then sell the bond.

- (a) What total cash will you receive from the bond after the 2½ years? Assume that periodic cash flows are reinvested at 12%.
- (b) If immediately after buying this bond, all market interest rates drop to 11% (including your reinvestment rate), what will be the impact on your total cash flow after 2½ years? How does this compare to part (a)?
- (c) Assuming all market interest rates are 12%, what is the duration of this bond?

**Solution:**

- (a) You will receive 160, reinvested that for 1.5 years, and 170 reinvested for 0.5 years. Then you will sell the remaining cash flows, discounted at 12%. This gives you:

$$160 \times (1.12)^{1.5} + 170 \times (1.12)^{0.5} + \frac{180}{1.12^{0.5}} + \frac{230}{1.12^{1.5}} = \$733.69$$

- (b) This is the same as part (a), but the rate is now 11%.

$$160 \times (1.11)^{1.5} + 170 \times (1.11)^{0.5} + \frac{180}{1.11^{0.5}} + \frac{230}{1.11^{1.5}} = \$733.74.$$

Notice that this is only \$0.05 different from part (a).

- (c) The duration is calculated as follows:

Year	1	2	3	4	Sum
Payments	160.00	170.00	180.00	230.00	
PV of Payments	142.86	135.52	128.12	146.17	552.67
Time Weighted PV of Payments	142.86	271.05	384.36	584.68	
Time Weighted PV of Payments	0.26	0.49	0.70	1.06	2.50
Divided by Price					

Since the duration and the holding period are the same, you are insulated from immediate changes in interest rates! It doesn't always work out this perfectly, but the idea is important.