

# Lab 13 - Moments of Inertia

Name \_\_\_\_\_

Partner's Name \_\_\_\_\_

## I. Introduction/Theory

The moment of inertia of a body can be determined as follows: The mass  $M$  whose moment of inertia is under study is attached to a vertical shaft of radius  $r$  by means of the moment of inertia rod. The vertical shaft is mounted on bearings which make it (and the mass  $M$ ) free to rotate when a torque is applied to it. A torque is supplied by a falling mass  $m$  which is connected to the shaft by means of a cord, one end of which is wrapped around the circumference of the shaft and the other end of which is attached to the mass.

If the mass  $m$  starts from rest and falls through a distance  $h$  in time  $t$ , potential energy of the amount  $mgh$  changes to kinetic energy. This kinetic energy consists of the translational energy of the falling mass  $\frac{1}{2}mv^2$  plus the rotational energy of the rotating system  $\frac{1}{2}I\omega^2$ ,

where  $v$  is velocity of falling mass,

$I$  is moment of inertia of rotating system,

$\omega$  is angular velocity of rotating system.

The initial potential energy is equal to the final kinetic energy,

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \quad (1)$$

Because the force of gravity is a constant, the mass  $m$  will be uniformly accelerated and its velocity  $v$ , time of fall  $t$ , and distance of fall  $h$  are related by,

$$v = 2h/t \quad \text{from } h = \frac{1}{2}at^2 \text{ and } v^2 = v_0^2 + 2ah \quad (2)$$

The velocity of fall  $v$ , the angular velocity  $\omega$ , and the radius of the vertical shaft  $r$  are related by,

$$v = r\omega \quad (3)$$

Equations 1, 2 and 3 can be combined and solved for  $I$ ,

$$I = mr^2(gt^2/2h - 1) \quad (4)$$

The moment of inertia of the rotating system  $I$  can be considered to be the sum of two parts,  $I_0 + I_1$  where  $I_0$  is the moment of inertia of the vertical shaft with rod ~~and wing nuts attached~~, and  $I_1$  is the moment of inertia of the load (including the wing nuts) attached to the rod.

## II. Equipment

Centripetal Force Apparatus, WL0930 configured for moment of inertia experiment

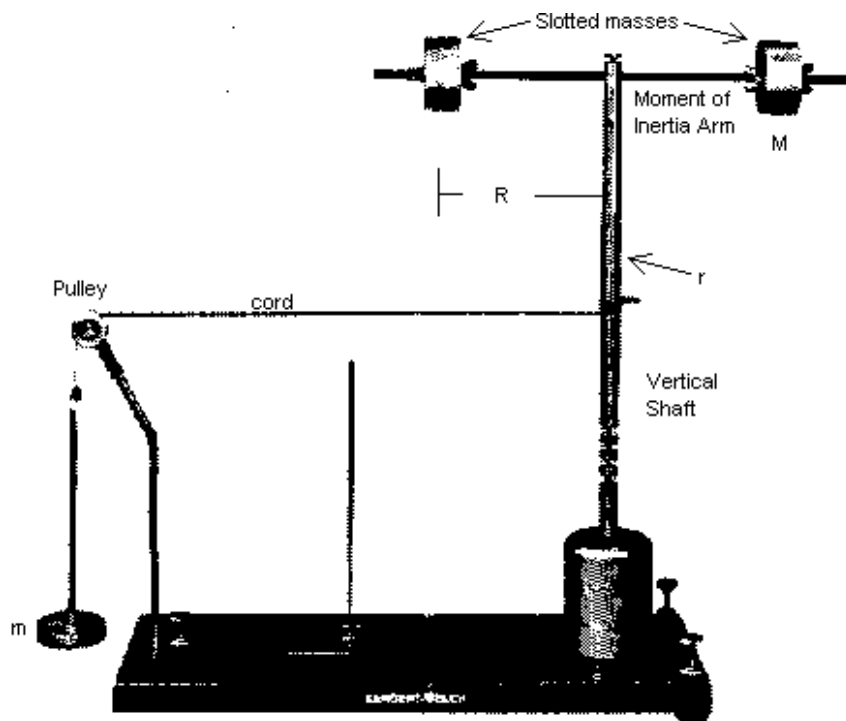
Slotted Weight Set

Vernier caliper

Stopwatch

Ruler and meter stick

Scale (500 g capacity)



### III. Procedure/Data

0. Use the wing nuts provided to secure a 100 gram slotted mass to each side of the rod about 12 cm from the center of the vertical shaft. Secure one end of a cord which is about 130 cm long to the screw in the vertical shaft and wind the cord *evenly* around the shaft, Pass the other end of the cord over the pulley, and attach a 100 gram mass to it.
1. Release the 100 gram mass and let it fall downward, causing the vertical shaft to rotate. Measure and record in Table 1, the following:
  - $h$  distance the accelerating mass falls;
  - $m$  mass of the accelerating (falling) mass;
  - $r$  radius of the vertical shaft;
  - $R$  distance of the slotted masses from the vertical shaft; and
  - $M$  mass of the slotted masses (including the wing nuts).
  - $t$  time for the mass to fall the distance  $h$ ;
2. Repeat the experiment several ( $\sim 5$ ) times to get a good value for  $t$  and  $S_t$ .
3. Remove the two 100 gram slotted masses and wing nuts from the moment of inertia arm, and again wind the cord; release the accelerating mass and measure the time  $t$  as per steps 1 and 2.
4. Use Equation 4 to calculate the moments of inertia of the rotating system with the additional mass ( $I$ ) and without the additional mass ( $I_0$ ). Subtract the two numbers to obtain the moment of inertia  $I_1$  of the load alone. Record results in Table 1.
5. Repeat steps 0, 1, 2 and 4 for two other values of the mass  $M$  (200 g and 300 g) with  $R$  at 12 cm. Plot  $M$  vs.  $I_1$  for constant  $R$  via Maple file `Lin_fit`. Make this plot after completion of the lab. Record results in Table 1.
6. Repeat steps 0, 1, 2 and 4 for two other values of  $R$  ( $\sim 8$  cm and a max value) with a fixed  $M$  (100 g). Plot  $R^2$  vs.  $I_1$  for constant  $M$  via Maple file `Lin_fit`. Make this plot after completion of the lab. Record results in Table 2.

7. When  $R$  is large compared to the dimensions of the object whose mass is  $M$ ,  $I_1$  is equal to  $MR^2$ . Using this relation, calculate the moment of inertia  $I_1$  for each value of  $M$  and  $R$  in steps 2, 4 and 5. What is the difference as a percent of the experimental value of  $I_1$ ?

	Step 1 and 2	Step 3	Step 5a	Step 5b
h				
m				
r				
R		XXXXXXXXXXXXX		
M		XXXXXXXXXXXXX		
t <sub>1</sub>				
t <sub>2</sub>				
t <sub>3</sub>				
t <sub>4</sub>				
t <sub>5</sub>				
t <sub>ave</sub>				
Std. Dev.				
Std. Err.				
I		I <sub>0</sub> =		
S <sub>I</sub>		S <sub>I0</sub> =		
	Step 4	XXXXXXXXXXXXX		
I <sub>1</sub>		XXXXXXXXXXXXX		
S <sub>I1</sub>		XXXXXXXXXXXXX		

Table 1. (Note: in the error propagation of  $I$ , consider only the uncertainty in time.)

Step 6	Input Step 1 and 2	Input Step 3	R ~ 8 cm	R at max value
h				
m				
r				
R		XXXXXXXXXXXXXX		
M		XXXXXXXXXXXXXX		
t <sub>1</sub>				
t <sub>2</sub>				
t <sub>3</sub>				
t <sub>4</sub>				
t <sub>5</sub>				
t <sub>ave</sub>				
Std. Dev.				
Std. Err.				
I		I <sub>0</sub> =		
S <sub>I</sub>		S <sub>I0</sub> =		
	Input Step 4	XXXXXXXXXXXXXX		
I <sub>1</sub>		XXXXXXXXXXXXXX		
S <sub>I1</sub>		XXXXXXXXXXXXXX		

Table 2 (note: second and third columns are copied from table 1!)

IV. Analysis

1. Tabulate the data for the plot in procedure step 65: I<sub>1</sub>, S<sub>I</sub>, and M (i.e. use all data having the same value for R to complete Table 1).
2. In Maple (lin\_fit.mws) plot the data from the previous step. Record slope and intercept. Attach Maple output to this lab via a staple in the upper left corner.



V. Conclusions (include physical concepts and principles investigated in this lab, independent of your experiments success, and summarize without going into the details of the procedure.)